MIMO detection employing Markov Chain Monte Carlo

V. Sundaram* and K.P.N.Murthy†

* *Department Electrical Engineering,*

University of Notre Dame,

IN46556 Indiana, United States of America

and

† *School of Physics, University of Hyderabad,*

Central University P.O.,

Gachibowli, Hyderabad 500 046,

Andhra Pradesh, India

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We propose a soft-output detection scheme for Multiple-Input-Multiple-Output (MIMO) systems. The detector employs Markov Chain Monte Carlo method to compute bit reliabilities from the signals received and is thus suited for coded MIMO systems. It offers a good trade-off between achievable performance and algorithmic complexity.

I. INTRODUCTION

Multiple-Input-Multiple-Output (MIMO) systems improve the channel capacity many-fold by the use of multiple antennas at transmitter and at receiver[1, 2]. It was shown [1] that the channel capacity increases linearly with the number of transmit antennas in a rich scattering environment. Because of this it has become possible to design transmission schemes where a single data stream is split into several substreams that are simultaneously transmitted to the available transmit antennas. MIMO forms the core technology for the next generation wireless networks, as seen in the standards IEEE 802.11n (wireless LAN) as well as IEEE 802.16e (wireless MAN).

The optimal MIMO receiver distinguishes the spatial signatures of different transmit substreams as seen at the receiver, while fully exploiting available receive diversity. The detection process also accounts for the structure of symbol constellations. The latter feature

implies that a Maximum *à* Posterior (MAP) receiver is inherently more robust (in terms of error rates) to low-rank channels, as noted in [3]. However, the implementation of an exact MAP detector requires testing of all possible hypotheses in order to compute the reliability of each bit, the number of hypotheses being exponential in the number of transmit antennas and the number of bits. There has therefore been extensive work both in reducing the computational complexity of optimal or near-optimal detectors [4, 5, 6] and in devising sub-optimal approaches to MIMO detection. Some of the latter include a space-time DFE with hard [7] and soft cancellation[8] and the use of iterative receivers [9, 10]. The major drawback of all the sphere decoding algorithms and their variants is their worst case complexity (which is exponential) and the problems encountered in computing soft values.

Markov Chain Monte Carlo (MCMC) methods essentially consist of drawing samples from a desired probability distribution. Multidimensional systems (such as MIMO) are specially suited for MCMC methods, whose complexity is at most polynomial with respect to signal dimensions. In a recent paper, Guo and Wang [11] proposed detection methods for MIMO systems based on sequential Monte Carlo (SMC) method. Dong, Wang and Doucet [12] applied the same method to a BLAST-type receiver and demonstrated that this can significantly improve the performance of MIMO detectors. Recently, Berouzhny *et al* [13] have reported improved performance using a Gibbs sampler. In this paper, we develop a Soft-In-Soft-Out (SISO) MIMO detector using an MCMC algorithm on a multidimensional lattice.

II. PROBLEM FORMULATION

Consider a MIMO system with transmit and receive antennas, see Fig. (1). The source bits are encoded and then interleaved before being mapped to M symbol streams for transmission. Each symbol stream contains symbols drawn from the constellation . For a flat-fading channel H, the received signal y is given by

$$\mathbf{y} = H\mathbf{s} + \mathbf{n} \qquad (1)$$

where $\mathbf{n}$ is the circularly symmetric channel additive Gaussian white noise vector, with independent components each with the same variance σ^2. For the k^{th} bit of the m^{th} symbol

hp

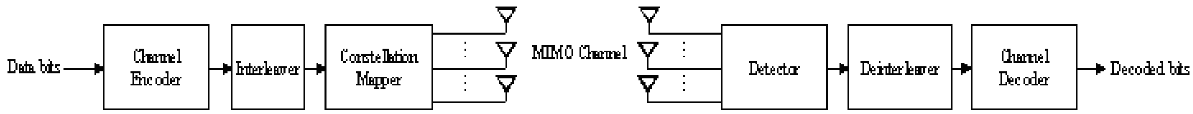

FIG. 1: MIMO system with transmit and receive antennas

s_m, denoted as $b_{m,k}$, the MAP detector computes its L^{th} -value given by

$$L_{m,k} = \log\left(\frac{p(b_{m,k}=1|\mathbf{y})}{p(b_{m,k}=0|\mathbf{y})}\right) \tag{2}$$

III. MARKOV CHAIN MONTE CARLO METHODS

Markov Chain Monte Carlo (MCMC) methods are a family of statistical simulation algorithms that help construct an ensemle of realizations or states from the desired multivariate probability distribution. In a typical MCMC method we start from an arbitrry state and construct a Makov chain whose asymptotic part contains states that belong to the desired distribution. This is ensured by suitably constructing a Markov transition matrix so that its stationary distribution coincides with the desired distribution. With easy availability of high performance computing machines in recent times this method has become popular and is being increasingly employed in a variety of fields that include physics, chemistry, biology, economics and engineering see *e.g.* [14].

In this paper, we use MCMC method to solve a Bayesian inference problem. To this end, we identify the desired probability distribution (say $\rho(x)$ of the random variable x) over which the inference is to be made. All possible discrete values that x can take, constitute the state space. We construct a Markov chain of states. The transition probability of the Markov chain determines the trajectory of the Markov Chain in the state space. Asymptoticlly the Markov chin converges to the desired disribution. The effectiveness of MCMC metod relies on the following:

a. the Markov Chain quickly reaches its stationary distribution and

b. the desired inference parameter usually depends only on the most probable values of the random variable, that is, those states that are most frequently visited by the Markov Chain.

Thus, spanning the entire state space that is usually exponential with the size of x is not necessary to extract useful information about the distribution.

IV. THE METROPOLIS ALGORITHM

The Metropolis algorithm [15] draws samples from a probability distribution $\rho(x)$, referred to as the target distribution in this paper. The idea behind this algorithm is to construct a Markov chain whose stationary distribution matches with the target distribution. Given the current state x a Metropolis sampler draws a candidate state, also called trial state x_t. The next state in the Markov chain can be either x itself or the trial state x_t. The choice is made randomly by drawing random numbers. To this end we define a Metropolis acceptance probabiity given by $p = \min(1, \rho(x)/\rho(x_t))$. If the random number drawn is less than p the trial state is accepted as the next entry in the Markov chain. Otherwise the current state continues and forms the next state. The transition probability matrix Q defined by the Metropolis algorithm, satisfies detailed balance condition, given by

$$\rho(x_i)Q(x_j|x_i) = \rho(x_j)Q(x_i|x_j) \quad \forall \;\; i,j \tag{3}$$

Note that the knowledge of the trget distribution is required only upto a normalization constant, which makes this algorithm very useful for simulating equilibrium statistical physics systems where the normalizing partition function is not known.

V. BIT RELIABILITY ESTIMATION USING MCMC

The posterior probability that $b_{m,k} = 1$ is givn by

$$p(b_{m,k} = 1|\mathbf{y}) = \sum_{S_m \in S_k^{(1)}} p(S_{m,k}|\mathbf{y}) \tag{4}$$

Here $S_k^{(1)}$ denotes all the symbols from the constellation S that have 1 in their k^{th} bit position. Expressing 4 as the marginalized probability over all other symbols

$$S^{(-m)} = [S_0 \; S_1 \; \cdots \; S_{m-1} \; S_{m+1} \; \cdots]$$

and applying Bayes theorem with prior symbol distribution

$$p(b_{m,k} = 1|\mathbf{y}) \; = \sum_{S_m \in S_k^{(1)}} \sum_{S^{(-m)}} p(S_m, \mathbf{S}^{(-m)}|\mathbf{y})$$

$$\propto \sum_{S_m \in S_k^{(1)}} \sum_{S^{(-m)}} p(\mathbf{y}|S_m, \mathbf{S}^{(-m)}|\mathbf{y}) p(S_m, \mathbf{S}^{(-m)})$$
$$\propto \sum_{S_m \in S_k^{(1)}} \sum_{S^{(-m)}} p(\mathbf{y}|S_m, \mathbf{S}^{(-m)})$$
$$\approx \sum_{\text{sinificant terms}} p(\mathbf{y}|S_m, \mathbf{S}^{(-m)}) \qquad (5)$$

The final expression in the above is obtained assuming the sum is dominated by those symbol vectors that are most probable. The number of these terms will be denoted by N_s. Eq. (5) can be interpreted as a Monte Carlo integral, see [13]. A similar expression can be derived for $p(b_{m,k} = 0|\mathbf{y})$.

VI. A RANDOM WALK METROPOLIS ALGORITHM ON A SYMBOL LATTICE

The L-values may thus be computed by a search over the symbol lattice. In a recent paper [13], a uniform Gibbs sampler is used to perform the search. The algorithm may be improved by confining the search to those lattice points that are more probable. This is captured by the likelihood $p(\mathbf{y}|\mathbf{S})$ that can be sampled using a Metropolis algorithm.

Consider a random walk Metropolis sampler that has the target distribution $p(\mathbf{y}|\mathbf{S})$. Observing that each vector $\mathbf{S}$ has a total of $4M$ neighbors (see Fig. 2),

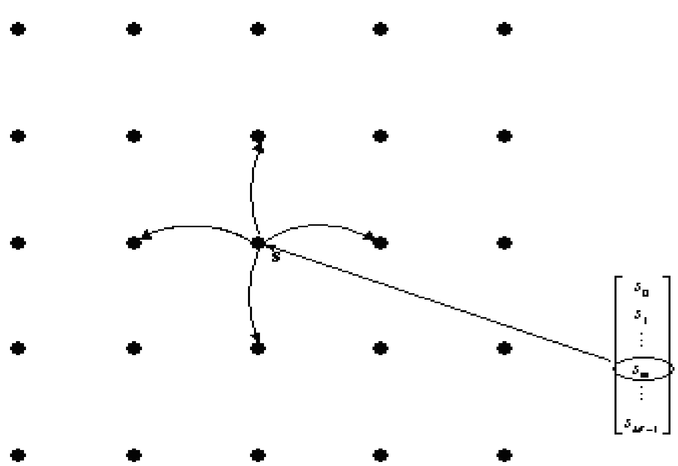

FIG. 2: Example of a nearest neighbour jump

$$Q(\mathbf{S}'|\mathbf{S}) = \begin{cases} \dfrac{1}{4M} & \text{if } \mathbf{S}' \text{ is a neighbour of } \mathbf{S} \\ \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

A periodic boundary condition is applied at the lattice edges. At each iteration step, a trial state is randomly chosen amongst the nearest neighbors of $\mathbf{S}$, and is accepted as the next state with the Metropolis acceptance probability

$$p = \min\left(1, \frac{p(\mathbf{y}|\mathbf{S}')}{p(\mathbf{y}|\mathbf{S})}\right)$$

VII. SIMULATION RESULTS

We now present some simulation results on a 3×3 MIMO system, *i.e.*, with 3 transmit and 3 receive antennas. A packet of 64 bytes was encoded using a rate-1/2 convolutional code with the generator matrix $G = [133, 171]$ (in octal notation). After interleaving, the bits are mapped to a (gray-coded) 16-QAM constellation. The MIMO channel is assumed to be i.i.d. Rayleigh fading, and changes with every channel use. At the receiver, the perfect CSI is assumed and the Metropolis sampler estimates the bit reliabilities (LLRs) using the Random Walk Metropolis Algorithm (RWMA) described above. The key parameters that determines the detector complexity is the number of Metropolis iterations (denoted by R), and the number of significant terms used (denoted by N_s). In all the simulations, we have set $N_s = 1$, that corresponds to the max-log-MAP approximation. We have also compared the results with uniform sampling. As the results in Figure 3 show, the frame error rate with the proposed algorithm is superior to that of uniform sampling, and seems to improve when R is increased.

The efficiency of a Metropolis sampler depends critically on its ability to efficiently span the sample space. This is captured in the acceptance ratio, which is defined as the number of number of actual state changes to the total number of Metropolis attempts. Efficient samplers have acceptance ratios between 0.4 to 0.7. The argument of the exponential function of the prior distribution is sclaled by a constant referred to in statistical physics literature as temperature. In several problems, the Metropolis algorithm can be improved by increasing the temperature. In the present case, noise plays the role of temperature in the system.

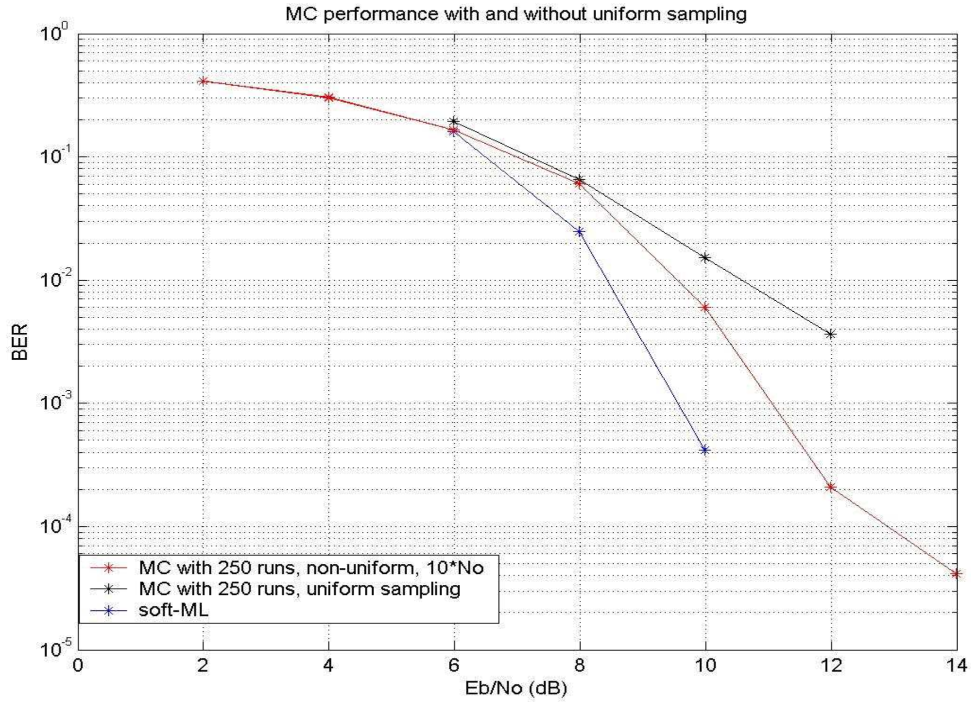

FIG. 3: Simulation of RWMA algorithm on a 3x3 16-QAM system

Increasing the temperature improves the sample space coverage. In our simulations, the sampling temperature was set to be 10 times that of the ambient noise.

VIII. CONCLUSIONS

We have proposed a soft output MIMO detector employing a random walk Metropolis algorithm. We find that this method provides a good tradeoff between computational complexity and soft output reliability. Some interesting possibilities are the inclusion of a prior in the detection process and reducing the complexity of each Metropolis step. These are currently being investigated.

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